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OPTIMAL INSTRUMENTATION OF STRUCTURES ON FLEXIBLE BASE FOR SYSTEM IDENTIFICATION

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SUMMARY

A criterion previously developed by Heredia-Zavoni and Esteva for selecting optimal sensor locations is used to analyse the optimal instrumentation of structures on soft soils. The stochastic response of a linear structural system on a flexible base is formulated for use of the criterion. The case of MDOF shear systems on flexible base, with uncertain lateral stiffness and subjected to random earthquake ground motions, is studied. The optimal location of accelerometers, the reduction of prior uncertainty on the lateral stiffness, the effects of the base flexibility, the relative influence of translation and rocking of the base, and the influence of recording noise are assessed and discussed. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: sensor location; optimal instrumentation; soil–structure interaction; Fisher information; random vibration; seismic response

INTRODUCTION

Records of the seismic response of full-size structures are of great value for identifying their mechanical properties and for improving our understanding of their dynamic behaviour. Sets of response records from sequences of earthquakes can also help us gain more knowledge about the process of damage accumulation and its influence on the reliability of structures. In different parts of the world, buildings have been instrumented to record their response to earthquake ground motions. In Mexico City, response records of some buildings have been used to identify modal frequencies, damping and shapes. Owing to the local soil conditions in downtown Mexico City, soil–structure interaction effects are important in the response of these buildings and should be taken into account when making decisions about their instrumentation.

Because of the limited number of sensors that may be available at a reasonable cost, criteria are necessary for selecting optimum sensor locations that yield the least uncertain estimates of the mechanical properties. Some solutions to the optimal sensor location problem have been proposed based on convergence and uniqueness criteria, and on the use of efficient estimators.[1–4] Recently, a criterion was proposed by Heredia-Zavoni and Esteva[5–7] for structures with

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uncertain properties subjected to random earthquake ground motions. The optimal location of the sensors minimizes the expected value of a Bayesian loss function expressed in terms of the Fisher information associated with the responses to be recorded. The criterion has been applied to the case of MDOF systems with uncertain lateral stiffnesses.

In this study, the criterion proposed by Heredia-Zavoni and Esteva is used to analyse the case of structures on soft soils where soil–foundation interaction effects should be taken into account. The paper first addresses the formulation of the response for the case of MDOF systems on flexible base subjected to stochastic earthquake ground motions; expressions are derived for the cross-spectral density function and the application of the criterion for optimal instrumentation is discussed. A case study of a MDOF shear structure with uncertain lateral stiffness resting on a flexible base is then presented. We study and analyse: (1) the optimal location of a single accelerometer; (2) the portion of prior uncertainty about the lateral stiffness that can be reduced by recording the building response; (3) the efficiency of placing an additional accelerometer at the base; (4) the effects of the base flexibility on the optimal instrumentation of the structure; (5) the distribution of information over the frequency content of the response; (6) the effect of recording noise on optimal instrumentation. The results are thoroughly discussed and conclusions and recommendations are given at the end.

EQUATIONS OF MOTION

Consider the multi-degree of freedom building system on flexible base shown in Figure 1; it is composed of a foundation or base and of a shear superstructure. The soil is assumed to be linearly elastic and soil–structure interaction due to translation and rocking of the base is taken into account by means of springs with translational and rotational stiffnesses, K_h, and K_θ, respectively. For the superstructure with Q levels shown in Figure 1, let $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ be the mass, damping and stiffness matrix, $\mathbf{H}$ a vector of the floor heights with respect to the base, $\mathbf{J}$ a vector of ones and $\mathbf{U}$ the response vector of contributions from the structures's deformation to the structure's displacements. Let m_0 and I_{m_0} denote the base mass and moment of inertia associated with its translation and rotation around an axis perpendicular to the plane of the structure, respectively. Figure 1 also shows the free-field horizontal ground displacement, v_g, and the horizontal displacement v_0 and rotation θ of the base due to soil–structure interaction.

The equations of motion for the system are:

$$\mathbf{M}\{\ddot{\mathbf{U}} + \mathbf{J}\ddot{v}_g + \mathbf{J}\ddot{v}_0 + \mathbf{H}\ddot{\theta}\} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{0} \tag{1}$$

$$\mathbf{J}^T\mathbf{M}\{\ddot{\mathbf{U}} + \mathbf{J}\ddot{v}_g + \mathbf{J}\ddot{v}_0 + \mathbf{H}\ddot{\theta}\} + m_0(\ddot{v}_g + \ddot{v}_0) + K_h v_0 = 0 \tag{2}$$

$$\mathbf{H}^T\mathbf{M}\{\ddot{\mathbf{U}} + \mathbf{J}\ddot{v}_g + \mathbf{J}\ddot{v}_0 + \mathbf{H}\ddot{\theta}\} + I_{m_0}\ddot{\theta} + K_\theta\theta = 0 \tag{3}$$

where superscript 'T' indicates a matrix transpose. The three equations of motion can be written in a more compact matrix form as follows:

$$\mathbf{M}^s\ddot{\mathbf{U}}^s + \mathbf{C}^s\dot{\mathbf{U}}^s + \mathbf{K}^s\mathbf{U}^s = -\rho\ddot{v}_g \tag{4}$$

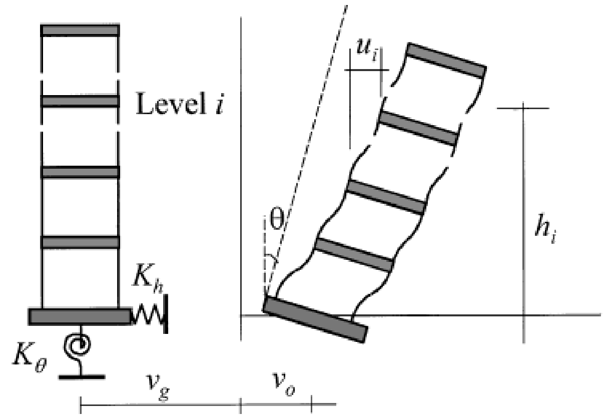

Figure 1. System on flexible base and displacement components

where

$$\mathbf{M}^{\mathrm{s}} = \begin{bmatrix} \mathbf{M} & \mathbf{MJ} & \mathbf{MH} \\ \mathbf{J}^{\mathrm{T}}\mathbf{M} & \mathbf{J}^{\mathrm{T}}\mathbf{MJ} + m_0 & \mathbf{J}^{\mathrm{T}}\mathbf{MH} \\ \mathbf{H}^{\mathrm{T}}\mathbf{M} & \mathbf{H}^{\mathrm{T}}\mathbf{MJ} & \mathbf{H}^{\mathrm{T}}\mathbf{MH} + I_{\mathrm{m}_0} \end{bmatrix}, \qquad \mathbf{C}^{\mathrm{s}} = \begin{bmatrix} \mathbf{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}^{\mathrm{T}} & 0 & 0 \\ \mathbf{0}^{\mathrm{T}} & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}^{\mathrm{s}} = \begin{bmatrix} \mathbf{K} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}^{\mathrm{T}} & K_h & 0 \\ \mathbf{0}^{\mathrm{T}} & 0 & K_\theta \end{bmatrix}, \qquad \mathbf{U}^{\mathrm{s}} = \begin{Bmatrix} \mathbf{U} \\ v_0 \\ \theta \end{Bmatrix} \tag{5}$$

and superscript s refers to the system on flexible base with $(Q+2)$ degrees of freedom; $\rho = \{\mathbf{MJ}, \mathbf{J}^{\mathrm{T}}\mathbf{MJ} + m_0, \mathbf{H}^{\mathrm{T}}\mathbf{MJ}\}$ is a $((Q+2)\times 1)$ vector.

Let φ_i, $i = 1, 2, \ldots, Q+2$, be the modal shapes of the system on flexible base and $\Phi = [\varphi_1\ \varphi_2 \ldots \varphi_{Q+2}]$ the modal matrix. Since $\mathbf{M}^{\mathrm{s}}$ and $\mathbf{K}^{\mathrm{s}}$ are symmetric and positive definite, the matrix products $\Phi^{\mathrm{T}}\mathbf{M}^{\mathrm{s}}\Phi$ and $\Phi^{\mathrm{T}}\mathbf{K}^{\mathrm{s}}\Phi$ result in diagonal matrices; let us assume that $\Phi^{\mathrm{T}}\mathbf{C}^{\mathrm{s}}\Phi$ is also a diagonal matrix. The solution to equation (4) can be written in terms of generalized co-ordinates as

$$\mathbf{U}^{\mathrm{s}} = -\sum_{i=1}^{Q+2} \Gamma_i \varphi_i x_i \tag{6}$$

where x_i satisfies the equation

$$\ddot{x}_i + 2\xi_i\omega_i\dot{x}_i + \omega_i^2 x_i = \ddot{v}_{\mathrm{g}} \tag{7}$$

and ω_i and ξ_i are the modal frequency and critical damping ratio, $c_i/m_i = 2\xi_i\omega_i$, $m_i = \varphi_i^{\mathrm{T}}\mathbf{M}^{\mathrm{s}}\varphi_i$, $c_i = \varphi_i^{\mathrm{T}}\mathbf{C}^{\mathrm{s}}\varphi_i$, $k_i = \varphi_i^{\mathrm{T}}\mathbf{K}^{\mathrm{s}}\varphi_i$, $\omega_i^2 = k_i/m_i$, and $\Gamma_i = L_i/m_i$ is the modal participation factor, $L_i = \varphi_i^{\mathrm{T}}\rho$. Since modes include the effects of foundation flexibility, the modal damping ratios indirectly include material soil damping. The total displacements along the translational degrees of freedom

of the system are given by

$$Y_j = v_g + \mathbf{r}_j^T \mathbf{U}^s, \quad j = 0, 1, 2, \ldots, Q \tag{8}$$

where $\mathbf{r}_j^T \mathbf{U}^s$ is the relative displacement of level j with respect to the free field ground motion. Vectors $\mathbf{r}_j$ $((Q+2)\times 1)$ are defined as: (i) For the superstructure $(j = 1, \ldots, Q)$:

$\mathbf{r}_j^T = [0 \quad \ldots \quad \underset{\text{level } j}{1} \quad \ldots \quad 0 \quad 1 \quad h_j]$ (storey height h_j is the j-th component of vector $\mathbf{H}$) and (ii) For the base displacement $(j = 0)$:

$\mathbf{r}_0^T = [0 \quad \ldots \quad 0 \quad \ldots \quad 0 \quad 1 \quad 0]$. Using equation (6) in equation (8) one obtains

$$Y_j = v_g + \sum_{i=1}^{Q+2} e_i^j x_i \tag{9}$$

where

$$e_i^j = -\Gamma_i \mathbf{r}_j^T \varphi_i \tag{10}$$

STOCHASTIC STRUCTURAL RESPONSE

Suppose now that the earthquake ground acceleration is modelled as a zero-mean, stationary, Gaussian random process. From equation (9), the cross-correlation function between displacement responses Y_i and Y_j, can be written as

$$\begin{aligned} R_{ij}(\tau) = {} & E[v_g(t+\tau)v_g(t)] + \sum_{p=1}^{Q+2} e_p^j E[v_g(t+\tau)x_p(t)] + \sum_{p=1}^{Q+2} e_p^i E[x_p(t+\tau)v_g(t)] \\ & + \sum_{p=1}^{Q+2}\sum_{q=1}^{Q+2} e_p^i e_q^j E[x_p(t+\tau)x_q(t)] \end{aligned} \tag{11}$$

and the associated cross-spectral density function of total response accelerations, $S_{ij}(\omega)$, is given by

$$S_{ij}(\omega) = S(\omega)\left[1 - \omega^2 \sum_{p=1}^{Q+2} e_p^j H_p^*(\omega) - \omega^2 \sum_{p=1}^{Q+2} e_p^i H_p(\omega) + \omega^4 \sum_{p=1}^{Q+2}\sum_{q=1}^{Q+2} e_p^i e_q^j H_p(\omega) H_q^*(\omega)\right] \tag{12}$$

where $S(\omega)$ is the spectral density function of the free-field ground acceleration, $H_p(\omega)$ is the modal transfer function

$$H_p(\omega) = \frac{1}{\omega_p^2 - \omega^2 + 2\mathrm{i}\xi_p\omega_p\omega}, \quad \mathrm{i} = \sqrt{-1} \tag{13}$$

and $H_p^*(\omega)$ is the complex conjugate.

CASE STUDY

Consider a shear building with uncertain lateral inter-storey stiffness resting on a flexible base with translational and rotational stiffnesses K_h and K_θ. Let K_m denote the uncertain lateral stiffness of any given inter-storey and $K_i = C_i K_m$ denote the stiffness of the ith inter-storey, where

C_i, $i = 1, 2, \ldots, Q$, are known constants. Let the frequency ratios a and b be defined as follows:

$$a = \frac{\omega_h}{\omega_\theta}, \qquad b = \frac{\bar{\omega}_1}{\omega_1} \tag{14}$$

where $\bar{\omega}_1$ is the fundamental frequency of the system on a rigid base, ω_1 is the fundamental frequency of the system on flexible base, and ω_h and ω_θ are the translational and rocking natural frequencies of the base, respectively,

$$\omega_h^2 = \frac{K_h}{\mathbf{J}^{\mathrm{T}}\mathbf{MJ} + m_0}, \qquad \omega_\theta^2 = \frac{K_\theta}{\mathbf{H}^{\mathrm{T}}\mathbf{MH} + I_{m_0}} \tag{15}$$

Using equation (15) in the definition of a in equation (14),

$$a^2 = \frac{\omega_h^2}{\omega_\theta^2} = \frac{K_h}{K_\theta} H_{\mathrm{eq}}^2 \frac{1 + \gamma_\theta}{1 + \gamma_h} \tag{16}$$

where the equivalent height $H_{\mathrm{eq}} = \sqrt{\mathbf{H}^{\mathrm{T}}\mathbf{MH}/\mathbf{J}^{\mathrm{T}}\mathbf{MJ}}$ can be thought of as the polar radius of the system with respect to its base, and $\gamma_h = m_0/\mathbf{J}^{\mathrm{T}}\mathbf{MJ}$, $\gamma_\theta = I_{m_0}/\mathbf{H}^{\mathrm{T}}\mathbf{MH}$. Let us assume that the fundamental mode shape of the superstructure on rigid base is equal to the fundamental mode shape of the superstructure on flexible base. It can be shown (see Appendix I) that,

$$\frac{1}{\omega_1^2} = \frac{1}{K_h}\frac{\alpha_1^2}{\bar{m}_1} + \frac{1}{K_\theta}\frac{\beta_1^2}{\bar{m}_1} + \frac{1}{\bar{\omega}_1^2} \tag{17}$$

where $\alpha_1 = \bar{\varphi}_1^{\mathrm{T}}\mathbf{MJ}$, $\beta_1 = \bar{\varphi}_1^{\mathrm{T}}\mathbf{MH}$, $\bar{m}_1 = \bar{\varphi}_1^{\mathrm{T}}\mathbf{M}\bar{\varphi}_1$, and $\bar{\varphi}_1$ is the fundamental mode shape of the superstructure on a rigid base. Substituting equation (16) for K_θ in equation (17) and using the definition of frequency ratio b in equation (14), one obtains that

$$K_h = \frac{\bar{\omega}_1^2}{\bar{m}_1(b^2 - 1)}\left(\alpha_1^2 + \frac{a^2}{H_{\mathrm{eq}}^2}\frac{1 + \gamma_h}{1 + \gamma_\theta}\beta_1^2\right) \tag{18}$$

In equation (18), the translational stiffness of the base K_h can be expressed in terms of the frequency ratios and of a series of parameters that only depend on the fundamental frequency and mode shape of the system on rigid base. The expressions in equations (16) and (18) define, in an implicit way, the stiffnesses of the base K_h and K_θ as a function of the lateral inter-storey stiffness of the superstructure for given values of the frequency ratios a and b. Notice that when b goes to one and the system approaches the rigid base condition, K_h and K_θ tend to infinity. The hypotheses in deriving equation (17) are reasonable when: (1) the base stiffness is larger than the lateral stiffness of the superstructure, and (2) the total mass of the superstructure and its moment of inertia with respect to the base are considerably greater than the mass and moment of inertia of the base. For the purpose of the example shown next, it is assumed that such hypotheses are valid. This reduces the uncertain system parameters to only the lateral stiffness of the superstructure K_m. Note, however that for the case where such hypotheses are not appropriate, for instance, very stiff low-rise buildings on soft soil, the methodology can be applied considering the base stiffnesses, K_h and K_θ, in addition to K_m, as the uncertain parameters to be identified.

Suppose that total accelerations along the response degrees of freedom of the structure are recorded. In the case of rigid base, since $K_i = C_i K_m$, $i = 1, 2, \ldots, Q$, the mode shapes will be independent of K_m and the square of modal frequencies will be directly proportional to K_m. As shown in equations (16) and (18) the base stiffnesses K_h and K_θ are also directly proportional to

K_m; then, the mode shapes of the system on flexible base, φ_i , are also independent of K_m, $\partial\varphi_i/\partial K_m = 0$ and $\partial e_i^j/\partial K_m = 0$. It follows from equation (12) that

$$\frac{\partial S_{ij}(\omega)}{\partial K_m} = S(\omega)\left[-\omega^2 \sum_{p=1}^{Q+2} e_p^j \frac{\partial H_p^*(\omega)}{\partial K_m} - \omega^2 \sum_{p=1}^{Q+2} e_p^i \frac{\partial H_p(\omega)}{\partial K_m} + \omega^4 \sum_{p=1}^{Q+2} \sum_{q=1}^{Q+2} e_p^i e_q^j \frac{\partial H_p(\omega)}{\partial K_m} H_q^*(\omega) \right.$$
$$\left. + \omega^4 \sum_{p=1}^{Q+2} \sum_{q=1}^{Q+2} e_p^i e_q^j H_p(\omega) \frac{\partial H_q^*(\omega)}{\partial K_m} \right] \quad (19)$$

Consider now the case of a five degree-of-freedom system. The height of the inter-storeys is equal to 120 in and masses at the floor levels are equal to 1 Klb-s^2/in. A 2 per cent critical damping ratio is taken for all of the modes. Experimental studies of instrumented buildings on soft soil in Mexico City have reported values for frequency ratio b in the range of 1·06–2·44, and values for a around 1·7–1·8.[8,9] From numerical studies on the influence of soil–structure interaction on the inelastic response of buildings, mean values for a and b equal to 2·4 and 1·14 have been obtained.[10,11] Here, a value of 2·0 is taken for a first and b is given values equal to 1·0, 1·3 and 2·0. The system is assumed to be subjected to a Gaussian ground acceleration with a Kanai–Tajimi spectral density function with characteristic frequency $\omega_g = 0{\cdot}8$ Hz and critical damping ratio $\xi_g = 0{\cdot}20$.

Let K denote the uncertain lateral stiffness of the first inter-storey and K_i denote the stiffnesses of the upper inter-storeys; we take $C_i = 1$ ($i = 2, \ldots, 5$) and the distribution of mean stiffness with height is uniform. Suppose a prior lognormal distribution for K with mean $\mu_k = 3000$ Klb/in and coefficient of variation COV = 0·15. Response records were assumed to be 10 s long and to have a sampling frequency of 50 Hz. In order to apply the criterion for optimal sensor location,[5–7] equations (12) and (19) are used to evaluate the covariances between Fourier coefficients of recorded responses. Then the Fisher information matrix is assembled and the expected Bayesian loss function is evaluated. The optimum array is that for which the expected loss function is a minimum. In this example, the expected values of the loss function were computed by means of Montecarlo simulations based on 3000 sample sets for K and K_i ($i = 2, \ldots, 5$). Except where indicated, the results shown next are for the case where only one accelerometer is available.

Figure 2 shows results in terms of an Uncertainty Reduction Index (URI) versus a normalized noise amplitude N_0/S_0 for $b = 1{\cdot}3$, where N_0 and S_0 are the white noise amplitudes of the recording noise and of the seismic motion at baserock level, respectively. The URI is the portion of the stiffness prior coefficient of variation that can be reduced by means of instrumentation, URI = (COV − COV$_f$)/COV, where COV$_f$ is the posterior coefficient of variation of the stiffness K associated with the process of recording the system response and then estimating the lateral stiffness, *before* any particular value of the response has been observed; it can be obtained since the posterior variance of K is equal to the expected value of the Fisher information inverse.[5] The results show that greater reductions of uncertainty are achieved when the accelerometer is placed at the fifth floor which then becomes the optimum location. Even for relatively large values of the normalized recording noise, placing the accelerometer at the top floor reduces prior uncertainty in at least 84 per cent. The *URI* when the accelerometer is placed at the other floors falls within those shown in Figure 2 for the first and fifth floors. Locating the accelerometer at the base reduces the prior uncertainty about the stiffness only for low levels of the normalized recording noise. Given that frequency ratio $a = 2$, the base is more flexible in rocking than in translation. These results indicate that the translational stiffness at the base is such that an accelerometer

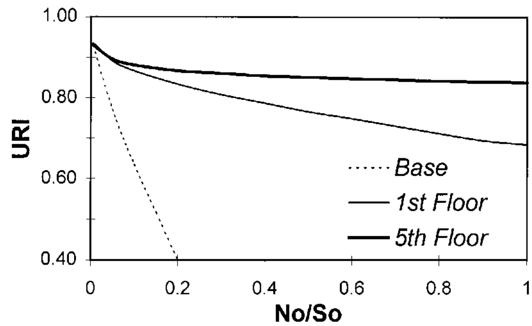

Figure 2. Uncertainty Reduction Index (URI); $b = 1{\cdot}3$

would record a signal similar to the free field ground acceleration with little information on the lateral stiffness of the superstructure.

Figure 3 compares the expected loss functions associated with recording the acceleration response of the first and the fifth floors for given values of frequency ratio b. For $b = 1{\cdot}3$, the fifth floor is the optimum location for the accelerometer for most of the normalized noise range. As the base becomes more flexible ($b = 2{\cdot}0$) the differences in the expected values of the loss functions are less significant. These results may suggest that as the flexibility of the base increases, the response of the superstructure with respect to the base decreases in importance in relation to that of the base itself and the amount of information obtained from recording any of the floor response accelerations is about the same.

Figure 4 shows: (a) the inverse variance of the Fourier coefficients A_k as a function of frequency ω_k, $1/E[A_k^2]$; (b) the sensitivities of such recorded response variances to the lateral stiffness K, $\partial E[A_k^2]/\partial K$, and (c) the contribution to the Fisher information from the recorded response components at each frequency ω_k, $M_k = \frac{1}{2}[\mathbf{C}_k^{-1}\partial\mathbf{C}_k/\partial K]^2$. Consider first the case where $b = 1{\cdot}3$. Peak values for the variance of the recorded response Fourier coefficients are at the first and second mean modal frequencies (Figure 4(a)). A local minimum for the inverse variance also occurs at the ground motion characteristic frequency $\omega = 5$ rad/s, which is related to the expected large amplitudes of those acceleration response components at frequencies close to the ground motion dominant ones. The upper boundary in the inverse variances at about $5\ \mathrm{s}^4/\mathrm{in}^2$ for frequencies in the range of 20–35 rad/s is set by the level of recording noise. Figure 4(b) shows that the variance of the Fourier coefficients in the acceleration records are sensitive to the lateral stiffness at frequencies around the mean fundamental frequency of the system. Notice that the variances for the acceleration Fourier coefficients recorded at the fifth floor are much more sensitive than those for the accelerations recorded at the first floor. The contributions to the Fisher information in the acceleration records come from response components at frequencies around the mean fundamental frequency of the system (Figure 4(c)). The Fisher information is proportional to the area under the curves shown in Figure 4(c). The Fisher information associated with recording the fifth floor response acceleration is greater than that for the first floor response, which is consistent with the results shown in Figure 2 in terms of uncertainty reduction. The significantly greater sensitivity of the variances associated with recording the fifth floor response acceleration, compared to that of the first floor response, is dominant in the computation of the

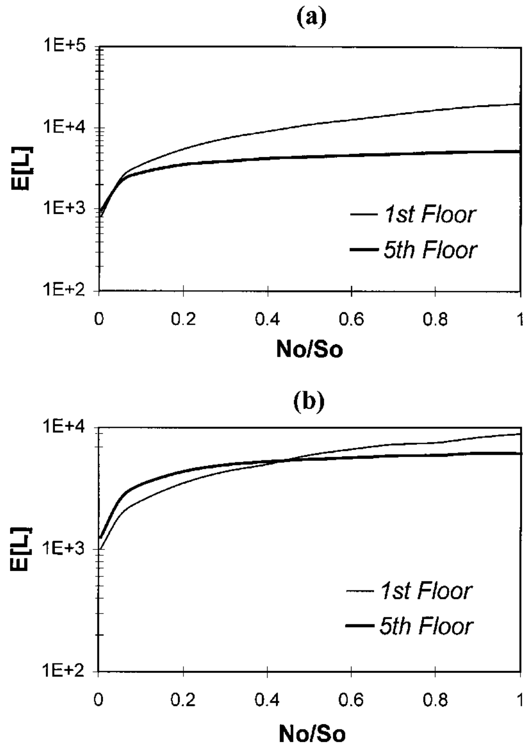

Figure 3. Expected values of Bayesian loss function, $E[L]$: (a) $b = 1{\cdot}3$; (b) $b = 2{\cdot}0$

Fisher information and it is mainly due to such sensitivity that the fifth floor turns to be the optimal location for the accelerometer.

Consider now the variations with frequency ratio b. Figure 4(a) shows that as the base becomes more flexible and the mean fundamental frequency approaches the value of the ground motion characteristic frequency, the recorded response variances increase. Figure 4(b) shows that both the first and fifth floor response variances are more sensitive to changes in the lateral stiffness when the base is more flexible. Not that the fifth floor response variance is much more sensitive to the lateral stiffness of the building than the response variance of the first floor for all cases of flexible base considered. When recording the first floor response, the values of information increase with frequency ratio b at a faster rate than do the values when the fifth floor response is recorded (Figure 4(c)). Notice that in both cases the band of response frequencies that contribute to information narrows as the base becomes more flexible. For the fifth floor response, the total amount of information decreases as the base becomes more flexible, indicating that the response variance increases with frequency ratio b at a faster rate than its sensitivity. In case of the first floor response, the fact that the total amount of information increases with frequency ratio b, indicates that the response variance does not increase fast enough compared to the changes in its sensitivity. Overall, the total amount of information obtained from recording any of these floor responses tends to be about the same when the base becomes more flexible, possibly suggesting that the superstructure approaches a type of rigid body response.

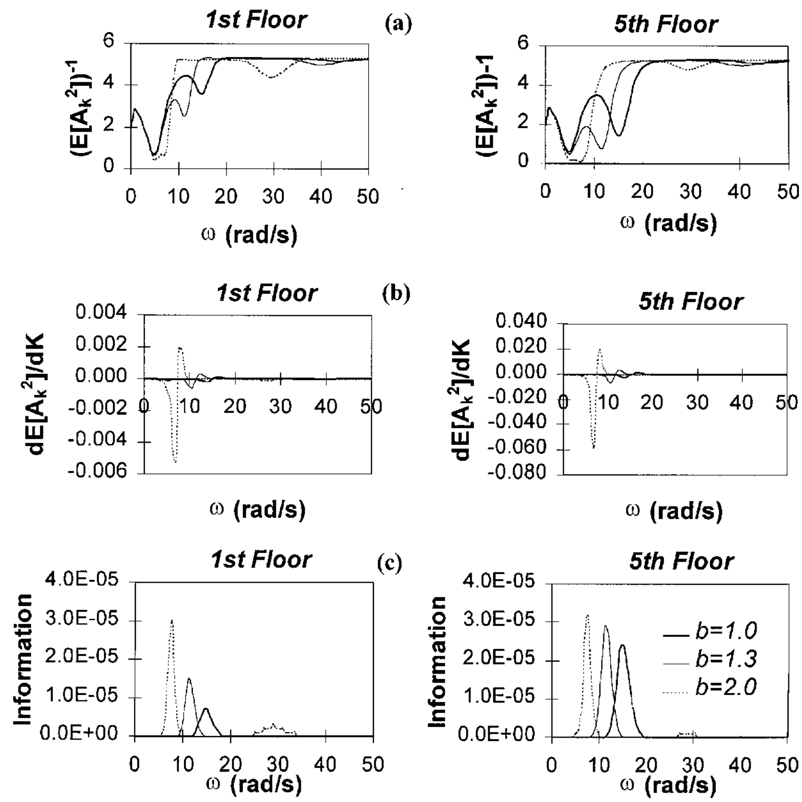

Figure 4. Frequency domain analysis; $N_0/S_0 = 0{\cdot}60$: (a) inverse variance of Fourier coefficients A_k; (b) variance sensitivity of Fourier coefficients A_k; (c) information

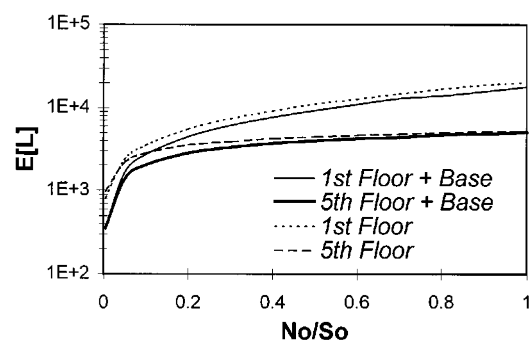

Figure 5. Expected values of Bayesian loss function, $E[L]$; $b = 1{\cdot}3$

Consider now the case where an additional accelerometer is placed at the base. Figure 5 shows the expected loss function versus N_0/S_0 for $b = 1{\cdot}3$. As seen, more information is not gained by placing an additional accelerometer at the base. Since frequency ratio $a = 2{\cdot}0$ the base is more flexible in rocking than in translation. The results suggest that soil–structure interaction is mainly

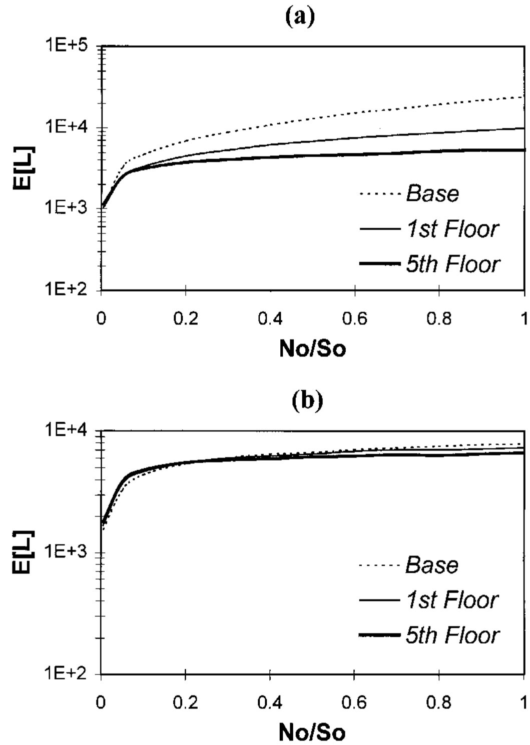

Figure 6. Expected values of Bayesian loss function, $E[L]$; $a = 0{\cdot}5$: (a) $b = 1{\cdot}3$; (b) $b = 2{\cdot}0$

due to rocking and that recording the translational acceleration of the base is probably equivalent to recording the free-field ground motion.

The effects of a more flexible translational spring at the base are analysed next. Results were obtained for a frequency ratio $a = 0{\cdot}5$, so that the translational frequency of the base is half that of rocking. Figure 6 shows the uncertainty reduction index when a single accelerometer is placed at the fifth, first or base levels. The fifth floor is the optimum location for the accelerometer; however, a great deal of prior uncertainty on the lateral stiffness can also be reduced if the first floor response is recorded. Notice that recording the base response now yields enough information to achieve uncertainty reductions of at least 65 per cent. In contrast to the case considered above (Figure 2), the base is flexible in translation relative to rocking and recording its accelereration seems to give enough information on the lateral stiffness of the superstructure to achieve such reductions of prior uncertainty. As the base becomes more flexible and the system approaches a rigid body response, one can expect similar reductions of prior uncertainty when recording any of the response accelerations.

CONCLUSIONS

Several issues related to the instrumentation of buildings on flexible base have been addressed in this paper. An expression for the cross-spectral density function of the response of MDOF

systems on flexible base, where soil–structure interaction is modeled by means of translational and rotational springs, was derived and the criterion proposed by Heredia-Zavoni and Esteva for selecting the optimal location of sensors was used. The case of shear structures with uncertain perfectly correlated lateral stiffness was studied.

An example of a five degree-of-freedom structure was considered. The results showed that a significant amount of prior uncertainty about the lateral stiffness can be reduced by placing a single accelerometer at the top floor, even for relatively large amplitudes of recording noise. The main contributions to the Fisher information in the recordings come from response components at frequencies around the mean fundamental frequency of the system. The sensitivity of the response acceleration recorded at the top floor to the lateral stiffness is considerably higher than that of other floor responses; it is mainly due to such sensitivity that the top floor turns to be the optimal location for a single accelerometer. As the base becomes flexible, the amount of information in the acceleration records from other floor responses approaches that of the top floor, suggesting a type of rigid body response for the superstructure. The relative influence of translation and rocking of the base was assessed. Quantitative results were given that allow assessing the relative influence of rocking and translation of the base; when soil–foundation interaction effects are mainly due to rocking, results showed that there is not a significant further reduction of prior uncertainty if an additional accelerometer is placed at the base.

Different conclusions may be obtained when making decisions about structures with lateral stiffnesses that are not perfectly correlated, or when other properties such as damping are to be identified. Also, a different response formulation is needed when it is not reasonable to asssume that the mass and moment of inertia of the base are small compared to those of the superstructure. The optimizing criterion originally proposed by Heredia-Zavoni and Esteva can still be used to deal with such issues. For non-linear response, the optimal location problem can be stated in terms of the efficient assessment and monitoring of damage accumulation. These topics are currently under study by the authors and shall be addressed in future publications.

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APPENDIX I

Let us suppose that the base is massless. Setting $m_0 = 0$ and $\mathrm{I}_{\mathrm{m}_0} = 0$, then from equations (1)–(3)

$$K_h v_0 = \mathbf{J}^{\mathrm{T}}\mathbf{C}\dot{\mathbf{U}} + \mathbf{J}^{\mathrm{T}}\mathbf{K}\mathbf{U}, \qquad K_\theta \theta = \mathbf{H}^{\mathrm{T}}\mathbf{C}\dot{\mathbf{U}} + \mathbf{H}^{\mathrm{T}}\mathbf{K}\mathbf{U} \tag{20}$$

Substituting equation (20) in equation (1)

$$\mathbf{M}\left[\ddot{\mathbf{U}} + \mathbf{J}\ddot{v}_{\mathrm{g}} + \frac{\mathbf{J}}{K_h}\{\mathbf{J}^{\mathrm{T}}\mathbf{C}\dddot{\mathbf{U}} + \mathbf{J}^{\mathrm{T}}\mathbf{K}\ddot{\mathbf{U}}\} + \frac{\mathbf{H}}{K_\theta}\{\mathbf{H}^{\mathrm{T}}\mathbf{C}\dddot{\mathbf{U}} + \mathbf{H}^{\mathrm{T}}\mathbf{K}\ddot{\mathbf{U}}\}\right] + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{0} \tag{21}$$

For the case of undamped free vibration (21) can be written as

$$\mathbf{M}_{\mathrm{F}}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{0} \tag{22}$$

where

$$\mathbf{M}_{\mathrm{F}} = \mathbf{M}\left[\mathbf{I} + \frac{1}{K_h}\mathbf{J}\mathbf{J}^{\mathrm{T}}\mathbf{K} + \frac{1}{K_\theta}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{K}\right] \tag{23}$$

The eigenvalue problem $[\mathbf{K} - \omega^2\mathbf{M}_{\mathrm{F}}]\varphi_e = \mathbf{0}$ yields the modal frequency and shape, ω and $\varphi_e(Q \times 1)$, of the superstructure for the system on a flexible massless base. For the superstructure on rigid base, the eigenvalue problem $[\mathbf{K} - \bar{\omega}^2\mathbf{M}]\bar{\varphi} = \mathbf{0}$ yields the corresponding modal frequency and shape, $\bar{\omega}$ and $\bar{\varphi}(Q \times 1)$. Under the hypothesis that for the fundamental mode shapes, $\varphi_{e1} = \bar{\varphi}_1$, then

$$\frac{1}{\omega_1^2} = \frac{\bar{\varphi}_1^{\mathrm{T}}\mathbf{M}_{\mathrm{F}}\bar{\varphi}_1}{\bar{\varphi}_1^{\mathrm{T}}\mathbf{K}\bar{\varphi}_1} \tag{24}$$

Substituting equation (23) in equation (24) we have that

$$\frac{1}{\omega_1^2} = \frac{1}{K_h}\frac{(\bar{\varphi}_1^{\mathrm{T}}\mathbf{MJ})(\mathbf{J}^{\mathrm{T}}\mathbf{K}\bar{\varphi}_1)}{\bar{\varphi}_1^{\mathrm{T}}\mathbf{K}\bar{\varphi}_1} + \frac{1}{K_\theta}\frac{(\bar{\varphi}_1^{\mathrm{T}}\mathbf{MH})(\mathbf{H}^{\mathrm{T}}\mathbf{K}\bar{\varphi}_1)}{\bar{\varphi}_1^{\mathrm{T}}\mathbf{K}\bar{\varphi}_1} + \frac{\bar{\varphi}_1^{\mathrm{T}}\mathbf{M}\bar{\varphi}_1}{\bar{\varphi}_1^{\mathrm{T}}\mathbf{K}\bar{\varphi}_1} \tag{25}$$

Since for the rigid base case, $\mathbf{K}\bar{\varphi}_1 = \bar{\omega}_1^2\mathbf{M}\bar{\varphi}_1$, then from equation (25),

$$\frac{1}{\omega_1^2} = \frac{1}{K_h}\frac{(\bar{\varphi}_1^{\mathrm{T}}\mathbf{MJ})^2}{\bar{\varphi}_1^{\mathrm{T}}\mathbf{M}\bar{\varphi}_1} + \frac{1}{K_\theta}\frac{(\bar{\varphi}_1^{\mathrm{T}}\mathbf{MH})^2}{\bar{\varphi}_1^{\mathrm{T}}\mathbf{M}\bar{\varphi}_1} + \frac{1}{\bar{\omega}_1^2} \tag{26}$$

Let $\alpha_1 = \bar{\varphi}_1^{\mathrm{T}}\mathbf{MJ}$, $\beta_1 = \bar{\varphi}_1^{\mathrm{T}}\mathbf{MH}$, $\bar{m}_1 = \bar{\varphi}_1^{\mathrm{T}}\mathbf{M}\bar{\varphi}_1$; then equation (26) can be written as follows:

$$\frac{1}{\omega_1^2} = \frac{1}{K_h}\frac{\alpha_1^2}{\bar{m}_1} + \frac{1}{K_\theta}\frac{\beta_1^2}{\bar{m}_1} + \frac{1}{\bar{\omega}_1^2} \tag{27}$$

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